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Erratum

Erratum to "Varieties of simplicial groupoids I: Crossed complexes" [J. Pure Appl. Algebra 120 (1997) 221–233][1]

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Daniel Conduché has pointed out to us a slip in the calculation of the verbal subgroupoid that starts on p. 230 of [4]. The application of Lemma 3.2. on that page to the calculation does not take into account the intersection used to get to the Moore complex in dimensions higher than 1. He noted, for instance, that the denominator of $D(\phi)_2$ is too small, since it should also contain $[s_0NH_1, s_1NH_1]$, which is certainly killed in $D(\phi)_2$ but does not seem to be derivable from the terms in the given denominator. A similar error occurs in higher dimensions and so the description of $ND(\phi)_n$ as the quotient of NH_n by $[NH_n, K_n]$ where $K_n = Ker(\phi_n : H_n \to \pi_0(H_.))$ is incorrect.

The argument is fairly easily corrected to get a result that is in some ways nicer and simpler than the incorrect results 3.4 and 3.5! The replacement for Theorem 3.5 is now more easily stated directly in terms of $\mathcal{V}(G)$ itself rather than the Moore complex of this verbal subgroupoid.

Theorem 1 (Generating words for the variety of crossed complexes). *The verbal simplicial subgroupoid of $H_\bullet$ corresponding to the variety of groupoid T-complexes is given by*

(i) *$\mathcal{V}(H)_0$ is trivial,*

(ii) *$\mathcal{V}(H)_1$ is generated by all $[x,y], x \in NG_1, y \in Kerd_0$,*

(iii) *$\mathcal{V}(H)_n$ is generated by all $[x,y]$, $x \in Ker(d_{n-1} : H_n \to H_0)$, $y \in Ker(\phi_n : H_n \to G)$, for $n \geq 2$.*

The proof uses a result of Conduché [2, p. 158]. He introduced the ordered set $S(p) = \bigcup_r S(p,r)$, where $S(p,r)$ is the set of increasing surjections from $[p]$ to $[r]$. Each H_p can be filtered by a family of subgroups $H_{p,i}$, indexed by $i \in S(p)$. Each of

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these subgroups is a semidirect product of its successor and another subgroup of the form s_iNH_r (see [2, Lemma 1.1]). This gives a semidirect product decomposition of H_n as:

$$\left(\cdots\left((NH_n \rtimes s_{n-1}NH_{n-1}) \rtimes \cdots\right) \rtimes s_{n-1}s_{n-2}\ldots s_0NH_0,\right.$$

the order of the factors being that of the corresponding elements of $S(n)$. Now let $\boldsymbol{d}_{\boldsymbol{n-1}} : H_n \to H_0$ be the composite $d_0 \circ \cdots \circ d_{n-1}$. This epimorphism is split by $\boldsymbol{s}_{\boldsymbol{n-1}} = s_{n-1} \circ \cdots \circ s_0$ and corresponds to the outermost semidirect product. This gives us the split exact sequence to process through our analysis from [4]. Setting $\phi_0 : H_0 \to \pi_0(H) = G$, $\phi_n = \phi_0 \circ \boldsymbol{d}_{\boldsymbol{n-1}}$ and $N_n = Ker\boldsymbol{d}_{\boldsymbol{n-1}}$ gives, by Lemma 3.2. of our paper,

$$D(\phi)_n \cong \frac{N_n}{[N_n, N_n][N_n, \boldsymbol{s}_{\boldsymbol{n-1}}(d_0NH_1)]} \oplus \boldsymbol{s}_{\boldsymbol{n-1}}(D_{\phi_0}).$$

Naturality will now decompose the first of these factors into a direct sum of Dold–Kan factors (corresponding to the semidirect factors above), but note, and here is where our slip occurred, in abelianising a semidirect product, you have to kill the action as well as abelianising the factors themselves. Thus in abelianising N_2, for instance, the decomposition

$$N_2 \cong ((NH_2 \rtimes s_1NH_1) \rtimes s_0NH_1)$$

means that we must kill the action of s_0NH_1 on s_1NH_1, and thus must divide out by the subgroup $[s_0NH_1, s_1NH_1]$ (amongst others), but this is actually a subgroup of the factor NH_2. In general for any $\boldsymbol{i},\boldsymbol{j} \in S(n)$, with $\boldsymbol{i} < \boldsymbol{j}$ in the ordering, we will need to kill a term of the form $[s_iNH_{n-r}, s_jNH_{n-s}]$. This may well be in NH_n as shows recent work of Mutlu, [6], see also [8].

To return to the proof, we note that $Ker\phi_n$ is the semidirect product of N_n and the subgroup, $\boldsymbol{s}_{\boldsymbol{n-1}}(d_0NH_1)$, which completes the argument.

A replacement for Corollary 3.4 of [4] is harder to give, as we have seen, however another approach solves the problem. The crossed complex, $C(H)$ associated with H is obtained from NH_n by factoring out $(NH_n \cap D_n)d_{n+1}(NH_{n+1} \cap D_{n+1})$ so giving a set of generators for this subgroup will do the trick. Again in Mutlu's thesis [6] there is given a generating set for each $NH_n \cap D_n$ (cf. [7, 9]). These generating elements are constructed by taking certain of the $[s_iNH_{n-r}, s_jNH_{n-s}]$ for suitable $\boldsymbol{i},\boldsymbol{j} \in S(p)$ and evaluating their projection into NH_n. The correspondence between the two approaches is thus very close. (The work of Carrasco and Cegarra [1] is closely related to these questions.)

A slight word of warning is needed. There are two forms of the Moore complex currently used; that using the intersection over all but the zeroth index (cf. Curtis [3]), and that using the intersection over all but the last face (cf. May [5]). The original paper was written using the first convention, but as the sources needed for this correction all use the second, we have followed that convention in this correction. As the two theories run completely in parallel, it is a simple matter to change from one to the other and this makes no difference to the statements of the result as that statement is independent of the convention used.

Acknowledgements

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